Non-Linear Mechanisms in the Brain*

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Z. Naturforsch. **53c,** 677–685 (1998); received April 29, 1998

Neuronal Models, Non-Linear Dynamics, Stochastic Resonance, Chaos, Emergence

Non-linear dynamical models of brain activity can describe the spontaneous emergence of large-scale coherent structures both in a temporal and spatial domain. We discuss a number of discrete time dynamical neuron models that illustrate some of the mechanisms involved. Of special interest is the phenomenon of spatio-temporal stochastic resonance in which coherent structures emerge as a result of the interaction of the neuronal system with external noise at a given level punitive data. We then discuss the general role of stochastic noise in brain dynamics and how similar concepts can be studied in the context of networks of connected brains on the Internet.

Introduction

It has been more than ten years that concepts based on non-linear dynamics, synergetics, attractors, and chaos have been introduced as alternatives to the traditional view of the brain as some form of digital computer (see Van Gelder, 1998 for a recent review of the arguments). In the groundbreaking paper "How brains make chaos in order to make sense of the world" (Skarda and Freeman, 1987) a number of arguments are presented why – in spite of the neuron/transistor analogy (as elementary switch and storage unit) – the brain processes information in fundamentally different ways. The basic new idea is that interacting neurons in the brain self-organize dynamically into coherently oscillating structures that are generated and recalled by input from external or internal sources. These structures are recognizable and reliably identify sensory input like different odors. But they cannot be characterized in simple terms of traditional, linear science. Freeman's group could show that the relevant sensory information is neither encoded in individual or groups of neurons nor in spectral information of their firing activity. Instead, they observed odor-specific changes in the erratic EEG signal from the olfactory bulb. The changes in the dynamical patterns are fast (< 100 ms) and strongly correlated with the stimulus and therefore identified with bifurcations of dynamical systems.

The terminology of non-linear dynamics in this context seems to be intuitive and certainly was helpful as a heuristic tool in modeling the observed phenomena. Nonetheless there have been a series of publications over the past ten years that more or less question the existence of "chaos" in the EEG -at least on a global scale (see e.g. Wright and Liley, 1996). At least part of the confusion in this discussion is due to some claims of experimental "proof" of low-dimensional chaos in EEG data (see e.g. the discussion in Layne *et al.* (1986) and Rapp (1993)). On the other hand, it is not very helpful to "disprove" the role of chaos in neuronal dynamics with the help of formal mathematical arguments. Many non-linear dynamical systems can display epochs of ordered behavior – where it can well be approximated by linear systems – alternating with bursts of chaos (Nicolis *et al.,* 1983) in a manner that the long-term average could be consistent with both periodic and chaotic attractors. Since we have no glimpse of an axiomatic brain theory in sight I am going to take an instrumentalist position and try to discuss domains and limita-

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[*] This communication is a contribution to the workshop on "Natural Organisms, Artificial Organisms, and Their Brains" at the Zentrum für interdisziplinäre Forschung (ZiF) in Bielefeld (Germany) on March 8–12, 1998.

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tions for the usefulness of dynamical systems concepts in describing EEG phenomena.[1]

Neural Networks and Their Interaction with Noise

Neuron models as iterated maps

There are quite a number of dynamical models in the literature that describe in more or less physiological detail the workings of a neuron. Their common characteristics is their "excitability" in the sense that their elements can undergo transitions to exited states either spontaneously or if their input values reaches a critical level. In the abstract formulation of the model of McCulloch and Pitts (1943) a neuron is represented by a simple binary switch that is coupled to all other elements with an intensity that is given by a matrix of weight functions. A vast literature in the artificial intelligence community describes how introducing smart algorithms for dynamically changing the "synaptic" weights allow these networks to "learn" and solve a growing number of practical problems.

Mathematically, the McCulloch and Pitts model is an example of a coupled map lattice where the generating map is discontinuous. In Bagley *et al.* (1986) we have introduced a piece-wise linear modification of this map that also can display more general dynamical features like bifurcations to self-sustained (periodic as well as chaotic) oscillations and refractory phases (Zbilut *et al.*, 1989; see also Appendix A for a brief description of the model.)

Global properties and noise

While it has been widely accepted that non-linearities play an essential role in the microscopic dynamics of neurons (Wright and Liley, 1996) there are still many questions about non-linear effects on global scales. Kaneko (1990) could show that his system of globally coupled, non-linear maps can exhibit global chaos (including large-scale coherent patterns) and thereby violate the law of large numbers and the limit theorem. These structures, however, will be destroyed (and the limit theorem restored) if large amounts of noise are added.

The argument has been made that arrays of deterministic, chaotic systems become extremely sensitive to any kind of perturbations and therefor cannot show robust and physiologically meaningful behavior (Chang *et al.* (1997) and references therein). Model simulations can become extremely susceptible to numerical artifacts. A similar situation is theoretically already given for simple, low-dimensional chaos that is typically structurally unstable. That means that for arbitrary small perturbations a chaotic system can become regular (non- chaotic). For that reason we have always emphasized the importance to study stochastic variants of chaotic systems (Haken and Mayer-Kress, 1981; Mayer-Kress and Haken, 1981). Especially in biological neural networks the background activities of distant neurons and other factors make stochastic non-linear models a plausible choice.

A model that supports large-scale, coherent, noise-sustained structures

In Jung and Mayer-Kress (1995a) we studied a version of the original McCulloch and Pitts model with exponential coupling (fixed in time) and constant refractory period. We were interested in the dynamics of large-scale structures in large arrays (400×400) and their interaction with noise. In order to exclude the influence of linear resonance phenomena due to reflecting or periodic boundary conditions we have in all our simulations used absorbing boundary conditions[2]. Under these conditions we expected that any coherent large-scale structures would be destroyed by the addition of stochastic noise. This was, however, not what was observed: For a sub-critical parameter range initial conditions can be found that lead to large-scale coherent, propagating wave fronts that would persist if the boundary conditions were not absorbing.

The addition of stochastic noise did not lead to the sudden destruction of the wave fronts but instead to a gradual increase of their curvature. For

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[1] Part of this work was completed while visiting the Institute for Medical Psychology and Behavioral Neurobiology of the University of Tübingen, Germany.

[2] Apparently boundary conditions of the neocortex cause a resonance effect that is responsible for the fact that frequencies of resting (alpha) EEG oscillations systematically decrease with increasing brainsize (see e.g. Nunez, 1996).

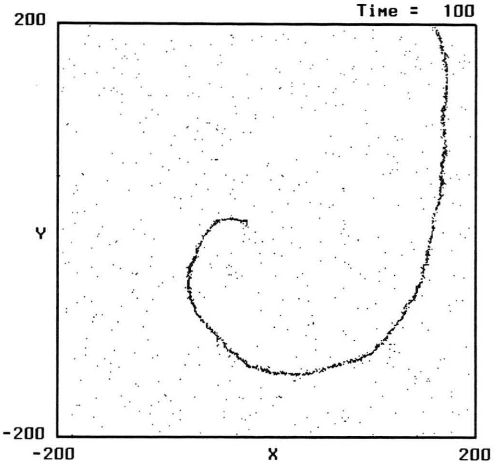

Fig. 1. Noise sustained spiral formation in modified McCulloch and Pitts array.

a certain range of parameters a single, noise-sustained spiral was formed. Since our model consists of elements with a finite number of states (resting, firing, refractory) it could be written as a cellular automaton (with rather messy update rules because of the exponential coupling term). At the above parameter configuration it would belong to the class IV category of cellular automata that are capable of universal computation "at the edge of chaos" (Wolfram, 1986; Langton, 1990).

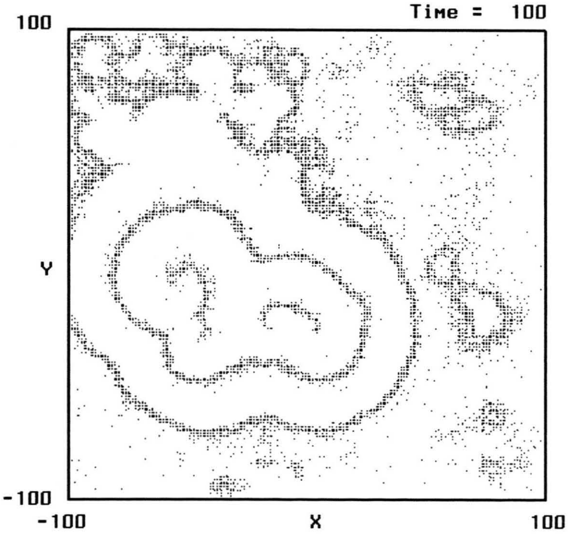

Fig. 2. Noise sustained spatio-temporal chaos in modified McCulloch and Pitts array.

For larger values of the noise level spatio-temporal chaos emerges whose average length scales decrease gradually with increasing noise level. This is consistent with the observations in (Mayer-Kress and Kaneko, 1989). We expect that in the limit of very large noise the central limit theorem will hold. But I think our simulations show that even in the presence of finite noise microscopic chaos can at least effect if not determine the global EEG. This is the case even under the condition of symmetric elements and constant coupling strength for all elements. We should also mention that in all of the parameter domains that we studied in these simulations the average firing rate of the elements was low.

Simulation of evoked response

Since our model showed a robust coupling of macro-structures to micro-dynamics in the presence of noise it seems natural to also study the response of our network to external stimuli. In Jung and Mayer-Kress (1995b) we could demonstrate that the local firing rate is dramatically increased in the presence of an external stimulus in the presence of noise at a well-defined level. The stimulus in that case consists of a bar that moves at a constant velocity across the array contributing to the input of the corresponding elements. The role of the noise was again critical: The response is selectively enhanced at a precisely determined critical noise level.

This phenomenon is the generalization of stochastic resonance to the situation of spatio-temporal arrays[3]. It is evident that for non-isotropic arrays selective response to movement direction can evolve. It is less obvious that this simple network can also exhibit velocity sensitive response characteristics: For a given set of system parameters and

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[3] Stochastic resonance is different from noise induced transitions between attractors (see e.g. Mayer-Kress and Haken, 1984) or in simulated annealing: In the latter the noise helps to select one attrator from a number of co-existing attractors namely the one associated with the minimum (generalized) potential energy. After the transition the system remains in the new attractor state even if the noise is turned off. In stochastic resonance, however, the stochastic force is an essential part of the dynamics: For instance there is no state in our model where large scale spirals would be sustained if the noise is turned off.

level of background noise the network shows a clear resonance in the response as a function of the stimulus velocity.

Based on these observations we would expect that natural selection has tuned neural parameters like coupling strength between neurons, propagation speed, refractory period, effective noise levels etc. to values that support efficient signal detection. Of course this tuning would have to be dynamic and could be directly related to the widely studied slow cortical potential (Birbaumer *et al.*, 1994). In a computational context one could say that brains have evolved to produce reliable results not only in spite of a noisy environment but that they even developed the capability to take advantage of the stochastic perturbations with the help of a form of spatio-temporal stochastic resonance. For the technical evolution of artificial neural nets this general concept might have far-reaching implications in nano-technology implementations where noise will play a much more significant role than in micro-electronics.

Control of chaos, non-linear resonance, and dynamical key

From the above discussion we can predict that the evoked response will display an intricate interaction between the momentary state of the brain (initial condition) and the neuronal representation of the stimulus. For example in Freeman's dynamical model of the olfactory bulb (Skarda and Freeman, 1987) we have the advantage that the sensory input is tightly coupled to the exhalation and inhalation phases of the breathing cycle. The perturbations from the receptor cells always reach the network of bulb neurons in the same dynamical state, in this example a chaotic background attractor. One would expect a very different response from the bulb if the odor stimulus were to arrive during a different phase of the breathing cycle for example by injecting the odor directly in the respiratory tract so that it would reach the bulb during exhalation.

In the more general case where sensory input from a stimulus is projected into different areas of the neo-cortex the situation may be much more complicated: We would expect a situation where a spatio-temporal perturbation interacts with spatio-temporal chaos. The response therefore would have a very broad statistical distribution. This is what is indeed observed in general evoked response experiments where the timing of the stimulus is determined by external parameters. Using one of Freeman's analogies this would be like a conductor starting at the scheduled time of the concert independent of the state of the orchestra.

A complicating factor is, however, that the observed signal consists of a superposition of background signal and the evoked response. According to estimates (Abeles, 1991; Aertsen, 1994; after Freeman (1996)) the modification of the variance of spontaneous background activity by induced (chaotic) dynamics is around one tenth of a percent. We think that this observation strongly supports the argument of triggering external stimuli by the state of spontaneous EEG. That implies that a domain of the observational state space needs to be defined (centered at a reference vector given by the signal values at all contributing sensors at a certain time point or sequence of time points (reconstructed state-space vector). Also, having control over selecting the background activity prior to the stimulus should improve signal-to-noise ratio through a better defined baseline.

If one takes the dynamical systems and chaos paradigm serious for the description of neocortical activity then much care needs to be applied to issues of initial conditions, timing, and relative phase (Mayer-Kress, 1994). The intricate interactions between non-linear oscillators and (optimal conditions for) external driving forces have been studied in the context of non-linear resonance and led to the concept of a "dynamical key" (Reiser *et al.*, 1987; Hubler, 1992). The basic idea is that any non-linear dynamical system will respond most sensitively to input that is closely matched to its intrinsic dynamics including transitions between ordered and chaotic behavior. We feel that experimental methods including non-linear resonance phenomena will play a growing role in future research. For example it might play a role in how we perceive music (Birbaumer *et al.*, 1996).

The general concept of "control of chaos" has been introduced in (Hubler, 1989; Ott *et al.*, 1990; Hubler, 1992) and applied in a growing number of different areas. From an engineering point of view it is often desirable to suppress chaotic fluctuations and induce a transition to more regular dynamics like that of a periodic limit cycle. This idea has been applied to electrical perturbation of animal brain dynamics. In Schiff *et al.* (1994) it could be shown that chaotic activity during induced epileptic seizures can be controlled in the sense that the activity becomes periodic. It is clear that this experiment is just a proof of principle and not intended for any clinical applications since periodic seizures are just as bad or worse than chaotic seizures.

For non-pathological situations we observe an irregular space-time dynamics involving different regions of the brain with a temporal evolution in the millisecond to second range. The corresponding nonlinear model would be that of spatio-temporal chaos. Any attempt of a direct control of this spatio-temporal chaos with electrical stimulation would require a large number of electrodes with a stimulus that follows a complex temporal pattern at each electrode.

There is an interaction between activity of the individual neurons and the coherent activity of the order parameter. This can be interpreted as "incomplete slaving" (Haken and Haken-Krell, 1997) which then leads to incoherent perturbations from the unslaved neurons in the form of quasi-stochastic noise. In the model simulations mentioned above we could show that arrays of coupled neuronal threshold elements can exhibit spatio-temporal stochastic resonance. That shows that the noise perturbations can play an important role in the information processing capabilities of the system through the interaction of two hierarchical levels.

Quantitative analysis of EEG/MEG data

Since more than twelve years ago methods from non-linear dynamics havo been used to analyze brain signals together or instead of standard statistical and spectral methods. Claims for evidence of low-dimensional chaos based on fractal dimension calculations (e.g. Babloyantz, 1986) have been questioned based on methodological and reproducability issues (e.g. Layne *et al.*, 1986; Rapp, 1993). Because of the intrinsic limitations of many biological signals especially in terms of stationarity it was soon recognized that a reliable estimate of typical dimensions of attractors in biological systems are not feasible. It appeared to be much more promising and reliable to study statistically significant changes in the (algorithm dependent) changes in the observed dimensional complexity (Mayer-Kress and Layne, 1987). Numerical simulations by Preissl *et al.* (1997) confirm that relative changes in the observed dimensionality remain consistent even for very short time-series (loss than one thousand data points).

This more practical approach – together with the interpretation of the observed (point-wise) dimension as a dynamical variable that probes geometric (as well as statistical) properties of attractors and their bifurcations (Mayer-Kress *et al.*, 1988) – led to a number of insights into brain dynamics that would have not been feasible with spectral or other traditional methods (see Elbert *et al.* (1994) for an extensive review).

The picture that emerges seems to indicate that the occurrence of narrow frequency band oscillations (alpha, gamma, or delta) is correlated with a reduction of dimensional complexity. This is consistent with dynamical systems theory where routes to chaos often are accompanied by limit cycle and torus attractors of lower dimension. A similar result could also be observed in a situation where a large number of low-amplitude oscillators synchronize to form a coherent order-parameter of lower dimension (Haken and Haken-Krell (1997) and references therein). If each of the entrained oscillators contributes to the EEG signal then one would predict an increase in the amplitude of the signal to be correlated with the reduction of observed dimensionality. This is indeed observed in the case of transitions to alpha or delta activity.

It would be very interesting to measure changes in dimensionality in experiments where enhanced gamma activity could be correlated with mental processes such as word/pseudo-word discrimination (Lutzenberger *et al.*, 1994). Since binding phenomena typically only last a small fraction of a second the associated oscillations of the corresponding cell assembly could consist of just a few oscillations (Skarda and Freeman, 1987). Therefore it might be more efficient to analyze the signal using wavelets instead of the standard Fourier transform (Mayer-Kress, 1994).

In order to discriminate between collections of linear oscillators and chaotic attractors one would also like to measure Lyapunov exponents (Mayer-Kress and Holzfuss, 1987) or at least obtain a semi-quantitative representation of local divergence rates from recurrence diagrams (Koebbe and Mayer-Kress, 1992). For the interpretation of dimensional changes in terms of neuronal cell assemblies it therefore would be desirable to simultaneously observe:

- dimension
- amplitude
- Lyapunov exponent/recurrence diagram
- frequency/wavelet spectrum

Music perception and resonance in EEG dimension

In this section we would like to discuss the interpretation of EEG in terms of cell assemblies for an experiment that showed an interesting resonance effect between the complexity of the stimulus (synthesized "musical" sound sequences) and the EEG dimension measured at a frontal location (Birbaumer *et al.*, 1996).

The sound sequences[4] consist either of modulated pitch ("melody") or inter-sound intervals ("rhythm"). The complexity of the sequences was evaluated according to standard dynamical systems criteria:

1. Four trials with periodic sequences (period 7 → period 5 → period 7 → period 5)
2. Four trials with intermittency type chaos (all four stimuli were generated in the intermittency region near period 5)
3. Four trials with quasi-random sequences (high-dimensional chaos) (all four stimuli were generated with uniformly distributed uncorrelated pseudo-random numbers, i.e.

$$x_{n+1} = a\, x_n \bmod 1,\ a \gg 1).$$

Since the periodic signal has a simple structure and the quasi-random sequences no recognizable structure at all we expected that they would have similar dimension values. The question is if we would expect an increase or decrease in dimension values for the chaotic sequence of intermediate complexity. If we expect that a larger number of assemblies need to be active in order to recognize recurring "themes" then we would predict an increase in dimension value. If we expect that the experience of aesthetic pleasure leads to a synchronization among assemblies then we would predict a decrease in dimension value. The experimental result was a decrease in dimension but not for all subjects: Subjects who had indicated a preference for classical music showed no significant change in dimension while listening to the rhythm sequences. Subjects who had indicated a preference for pop music showed no significant change in dimension while listening to the melody sequences.

These results[5] are very encouraging but our understanding of the connection between EEG dimensions and cell assemblies is only at the very beginning.

Generalizations of Dynamical Brain Paradigms

Organizations

Concepts from our understanding of neuronal processes in the brain have not only been generalized to machine learning and artificial intelligence. They also have been applied to knowledge processing in organizations – where organizations are seen as super-organisms. According to Radermacher (1997) knowledge processing is organized in four levels:

Signal → Feature → Symbol → Theory

Processing at each of these levels is performed by an increasingly integrated system of cells, assemblies and higher functional units and structures. Those neuronal sub-systems have been seen in analogy to similar functional structures in organizations and the corresponding concepts from self-organizing, complex systems and chaos have been applied to develop "fractal organizations".

Global brains

Extrapolating the process of evolution that has led to a hierarchy of organization with increasing complexity (from atoms to molecules to cells to brains) one can speculate about the next step: The Internet and other global computer networks have led to a global networking among human brains

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[4] The original sounds are available on our web site: http://www.ccsr.uiuc.edu/~gmk/Papers/MusicEEG/.

[5] The results from the data analysis have been confirmed by independent calculations under Dr. Skinner using his patented PD2I algorithm (Skinner, 1997; private communication).

to a degree that invites the analogy to a human brain (Mayer-Kress and Barczys, 1995). Emergent communication structures on the Internet with latencies well below 300 ms indicate that the analogy might lead to tools to solve global problems.

Appendix A: A Piecewise Linear, Discrete Time Model of a Chaotic Neouron

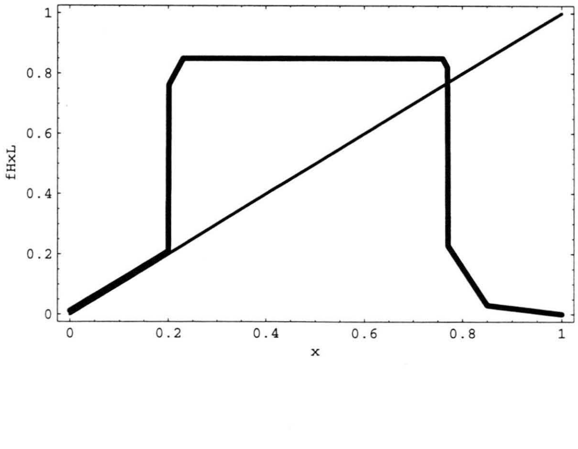
In the original McCulloch and Pitts model the neuronal state (axonal output can only take on values 0 or 1 depending on the sum of the dendritic inputs. We slightly generalized this model by allowing intermediate values for the output and an intrinsic dynamics represented by a one-dimensional difference equation ("map on the interval"), i.e. state x_n at time n will be mapped onto state $x_{n+1} = f(x_n)$ at time $n + 1$. The explicit form of the map is given (in self-explanatory Mathematica notation) as:

$$\begin{aligned}\mathrm{f}[x\_]{:}= \mathrm{Which}\,[&x < x0, .99(\,x - \mathrm{x}0) + .21,\\ &x < x1, a1\ (x - x0) + x2, x < x2, y2,\\ &x < x3, -3.1\ (x - x2) + a1\ (x1 - x0) +\\ &x2, x < y2, -2.5\ (x - x3) + x1,\\ &\mathrm{True}, yy3\ (1 - (x - y2)/(1 - y2))];\end{aligned}$$

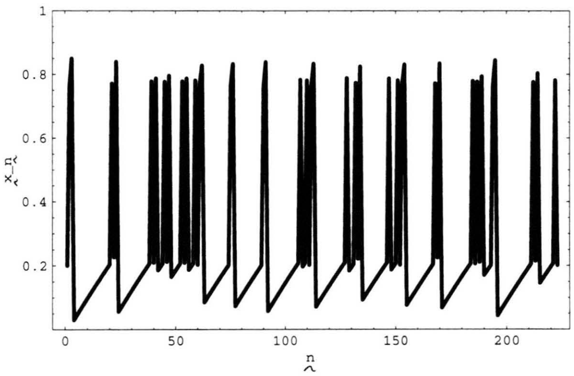
with parameters:

$$\begin{aligned}&x0 = .2;\ x1 = .23;\ x2 = .76;\ x3 = .77;\\ &a1 = 3;\ y2 = a1\ (x1 - x0) + x2;\ yy2 = x1;\\ &yy3 = -2.5\ (y2 - x3) + x1.\end{aligned}$$

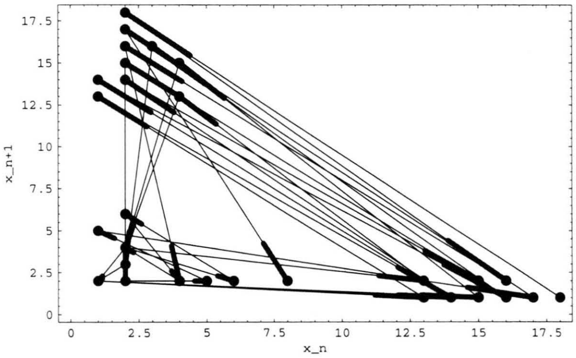
The graph of the function f is given by:



For the above parameter values the model generates the following spike-train:



The clustering of the data and transitions between the clusters can be visualized in a (x_{n+1}, x_n) state-space plot:



Acknowledgments

Stimulating discussions with A. Aertsen, N. Birbaumer, O. Roessler, H. Haken, W. Freeman, W. Lutzenberger, H. Preissl, K. Kaneko, F. J. Radermacher, W. Ray, O. E. Rossler, S. Skinner, and I. Tsuda are gratefully acknowledged. I would like to thank Prof. Birbaumer for his kind hospitality during my visit to his institute.

Abeles M. (1991), Corticonics. Cambridge U. Press, Cambridge, England.

Aertsen A. and Braitenberg V. (eds.) (1996), Brain Theory: Biological Basis and Computational Principles. Elsevier Science Publ., Amsterdam, New York.

Aertsen A., Erb M. and Palm G. (1994), Dynamics of functional coupling in the cerebral cortex: a model-based interpretation. Physica D **75,** 103–128.

Babloyantz A. (1986), Evidence of chaotic dynamics of brain activity during the sleep cycle. In: Dimensions and Entropies in Chaotic Systems (Mayer-Kress G., ed.). Springer-Verlag, Berlin, pp. 241–245.

Bagley R. J., Mayer-Kress G. and Farmer J. D. (1986), Mode locking, the Belousov- Zhabotinsky reaction, and one dimensional mappings. Phys. Lett. **114A,** 419–425.

Birbaumer N., Lutzenberger W., Rau H., Mayer-Kress G. and Braun C. (1996), Perception of music and dimensional complexity of brain activity. Intl. J. Bifurcations and Chaos **6**(2), 267–278.

Birbaumer N., Lutzenberger W., Elbert T. and Trevorrow T. (1994), Treshold variations in cortical cell assemblies and behavior. In: Cognitive Electrophysiology (Heinze H.-J., Münte T. F. and Mangun G. R., eds.). Birkhäuser, Boston, pp. 248–264.

ChangH.-J., Freeman W. J. and Burke B. C. (1998), Biologically modeled noise stabilizing neurodynamics for pattern recognition. Int. J. Bifurcation and Chaos **8** (2), 321–346.

Elbert T., Ray W. J., Kowalik Z., Skinner J. E., Graf K. E. and Birbaumer N. (1994), Chaos and physiology: deterministic cHaos in Excitable cell assemblies. Physiolog. Rev. **74**, 1–47.

Freeman W. J. (1995), Societies of Brains. Lawrence Erlbaum Assoc. Inc., Hillsdale/NJ.

Freeman W. J. and Barrie J. M. (1994), Chaotic oscillations and the genesis of meaning in cerebral cortex. In: Temporal Coding in the Brain (Buzsaki G. *et al.*, eds.). Springer, Berlin, 13–37.

Freeman W. J. (1996), Random activity at the microscopic neural level in cortex ("noise") sustains and is regulated by low-dimensional dynamics of macroscopic cortical activity ("chaos"). Int. J. Neural Systems **7**, 473–480.

Freeman W. J. and Jakubith S. (1993), Bifurcation analysis of continuous time dynamics of oscillatory neural networks. In: Brain Theory (Aertson A., ed). Springer-Verlag, Berlin.

Haken H., Haken-Krell M. (1997), Gehirn und Verhalten. DVA, Stuttgart.

Haken H., Mayer-Kress G. (1981), Chapman-Kolmogorov equation and path integrals for discrete chaos in the presence of noise. Z.Phys. B-Cond. Matter **43**, 183.

Hubler A. (1992), Modeling and control of complex systems: paradigms and applications. In: Modeling Complex Phenomena (Lam L., ed.). Springer, New York, pp. 5-65.

Hubler A. (1989), Adaptive control of chaotic systems. Helv. Phys. Acta **62**, 343.

Jung P. and Mayer-Kress G. (1995a), Noise controlled spiral growth in excitable media. CHAOS **5**, 458.

Jung P. and Mayer-Kress G. (1995b), Spatio-temporal stochastic resonance in excitable media. Phys. Rev. Lett. **74**(11), 2130–2133.

Kaneko K. (1990), Globally coupled chaos violates the law of large numbers. Phys. Rev. Lett. **65**, 1391–1394.

Koebbe M. and Mayer-Kress G. (1992), Use of recurrence plots in the analysis of time-series data. In: Nonlinear Modeling and Forecasting, SFI Studies in the Sciences of Complexity, Proc. Vol. **XII** (Casdagli M. and Eubank S., eds.). Addison-Wesley.

Langton C. G. (1990), Computation at the edge of chaos. Physica D **42**, 12–37.

Layne S. P., Mayer-Kress G. and Holzfuss J. (1986), Problems associated with dimensional analysis of electro-encephalogram data. In: Dimensions and Entropies in Chaotic Systems (Mayer-Kress G., ed.). Springer Series in Synergetics, Vol. **32**. Springer Verlag, Berlin, Heidelberg, New York, Tokyo, 246–256.

Lutzenberger W., Pulvermüller F. and Birbaumer N. (1994), Words and pseudo-words elicit distinct patterns of 30-Hz EEG responses in humans. Neurosci. Lett. **176**, 115–118.

Mayer-Kress G. and Barczys C. (1995), The Global Brain as an Emergent Structure from the Worldwide Computing Network, and Its Implications for Modeling, Tech. Rep. CCSR-94–22, The Information Society, Vol. **11**, No. 1, 1–28.

Mayer-Kress, G. (1994), Localized Measures for Non-Stationary Time-Series of Physiological Data , Integrative Physiological and Behavioral Science, July–September, Vol. **29**, No. 3, 203.

Mayer-Kress G., Barczys C., Freeman W. J. (1991), Attractor reconstruction from event-related multi-electrode EEG-data. In: Proceedings of the International Symposium "Mathematical Approaches To Brain Functioning Diagnostics" of the International Brain Research Organization (Holden, A. V., ed.). World Scientific.

Mayer-Kress G. and Hubler A. (1990), Time evolution of local complexity measures and aperiodic perturbations of nonlinear dynamical systems. In: Measures of Complexity and Chaos (Abraham N. B., Albano A. M., Passamante A. and Rapp P. E., eds.). Plenum, New York, 155–172.

Mayer-Kress G. and Kaneko K. (1989), Spatiotemporal chaos and noise. J. Stat. Phys. **54**, 1489–1508.

Mayer-Kress G., Yates F. E., Benton L., Keidel M., Tirsch W., Poppl S. J. and Geist K. (1988), Dimensional analysis of nonlinear oscillations in brain, heart and muscle. Mathematical Biosci. **90**, 155–182.

Mayer-Kress G. and Holzfuss J. (1987), Analysis of the Human Electroencephalogram with Methods from Nonlinear Dynamics, Proc. of Temporal Disorder in Human Oscillatory Systems (Rensing L., an der Heyden W. and Mackey M. C., eds.). Springer Series in Synergetics, Vol. **36**. Springer-Verlag Berlin, Heidelberg, 57–68.

Mayer-Kress G. and Layne S. P. (1987), Dimensionality of the human electroencephalogram. In: Perspectives in Biological Dynamics and Theoretical Medicine (Koslow S. H., Mandell A. J. and Shlesinger M. F., eds.). Ann. N.Y. Acad. Sci., Vol. **504**, 62–86, New York.

Mayer-Kress G. and Haken H. (1984), Attractors of convex maps with positive Schwarzian derivative in the presence of noise. Physica **10D**, 329–339.

Mayer-Kress G. and Haken H. (1981), The influence of noise on the logistic model. J. Stat. Phys. **26**, 149–171.

McCulloch and Pitts W. (1943), A logical calculus of the ideas immanent in nervous activity. Bull. Math. Biophys., vol. **5**, 115–133.

Morowitz H. J. and Singer J. (eds.) (1995), The Mind, the Brain, and Complex Adaptive Systems. Addison-Wesley, Reading/MA.

Nicolis J., Mayer-Kress G. and Haubs G. (1983), Nonuniform chaotic dynamics with implications to information processing. Z. Naturforsch. **38a**, 1157–1169.

Nunez P. L. (1996), Multiscale neocortical dynamics, experimental EEG measures, and global facilitation of local cell assemblies. Behav. Brain Sci. **19**, 305–306.

Ott E., Grebogi C. and Yorke J. (1990), Controling chaos. Phys. Rev. Lett. **64**, 1196.

Preissl H., Lutzenberger W., Pulvermuller H. and Birbaumer N. (1997), Fractal dimensions of short EEG time series in humans. Neurosci. Lett. **225**, 77–80.

Radermacher F. J. (1993), Cognition in systems. Cybern. Systems **27**, No. 1, 1–41.

Rapp P. (1993), Chaos in the neurosciences: cautionary tales from the frontier. Biologist **40**, 89–94.

Reiser G., Hübler A. and Lüscher E. (1987), Algorithm for the determination of the resonances of anharmonic damped oscillators. Z. Naturforsch. **42a**, 803.

Schiff S. J., Jerger K., Duong D. H., Chang T., Spano M. L.and Ditto W. L. (1994), Controlling chaos in the brain. Nature **230**, 615–620.

Skarda C. A. and Freeman W. J. (1987), How brains make chaos in order to make sense of the world. Behav. Brain Sci. **10**, 161–195.

Van Gelder T. (1998), The dynamical hypothesis in cognitive science. Behav. Brain Sci., to app.

Wolfram S. (1986), Theory and Applications of Cellular Automata. World Scientific, Singapore.

Wright J. J. and Liley D. T. J. (1996), Dynamics of the brain at global and microscopic scales: neural networks and the EEG. Behav. Brain Sci. **19**, 285–320, http://www.mhri.edu.au/bdl/papers/paper0/.

Zbilut J., Mayer-Kress G., Sobotka P., O'Toole M. and Thomas J. Jr., Bifurcations and intrinsic chaotic and 1/f dynamics in an isolated perfused rat heart. Biol. Cybernetics **61**, 371–378.